M: Mother Hulda

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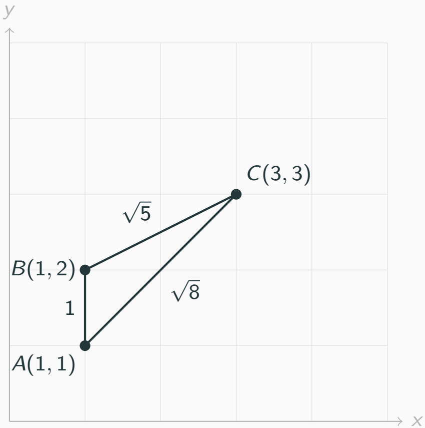
Problem

Given a matrix with squared pairwise distances, place n points in 2D with integer coordinates such that their distances are as in the matrix (or determine that this is impossible)

$$M = \begin{pmatrix} 0 & 1 & 8 \\ 1 & 0 & 5 \\ 8 & 5 & 0 \end{pmatrix}$$



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- Time complexity: $\mathcal{O}(k^2 \cdot n)$, and k is at most $\approx 10^2$.