

L: Lucky Hans

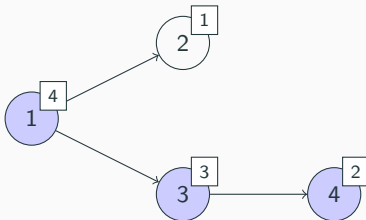
Problem author: Lucas Schwebler

Problem

- DAG on n vertices and m edges, vertex 1 can reach every other vertex.
- An integer k ($2 \leq n \leq 3000$, $1 \leq m \leq 9000$, $1 \leq k \leq n$).

Problem: Find a permutation p_1, \dots, p_N such that

- Topological order: For every edges $(u, v) \in E$, we have $p_u > p_v$
- Greedy path which starts at 1 and takes the maximum finishes at a vertex v with $p_v = k$



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Solution

- Label greedy path $1, \dots, v, u$
- Necessary conditions:

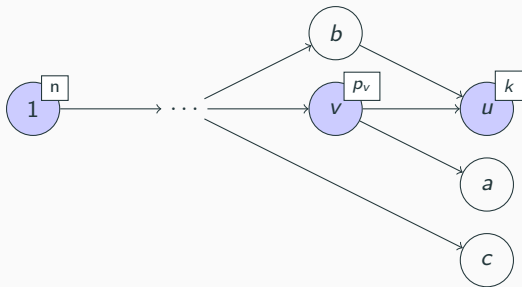


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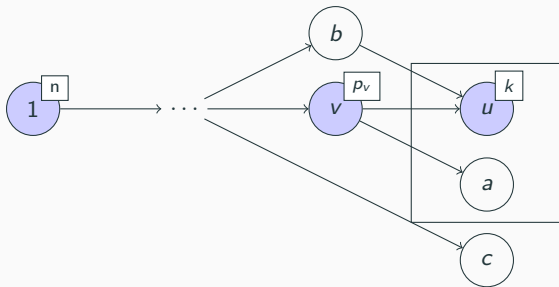


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Solution

- Label greedy path $1, \dots, v, u$
- Necessary conditions:
 - v can reach $\leq k$ vertices (excluding v)

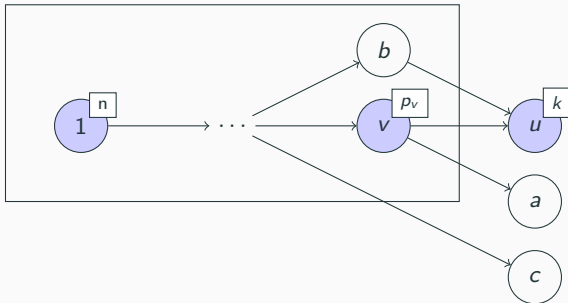


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Solution

- Label greedy path $1, \dots, v, u$
- Necessary conditions:
 - v can reach $\leq k$ vertices (excluding v)
 - u is reached by $\leq n - k$ vertices (excluding u)

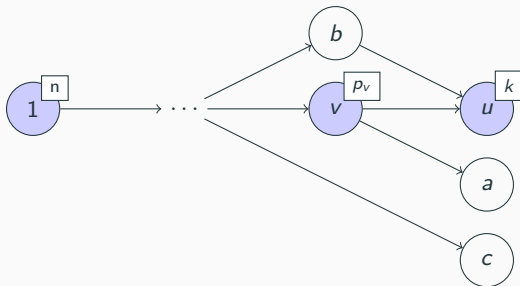


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- Label greedy path $1, \dots, v, u$
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- Also sufficient!



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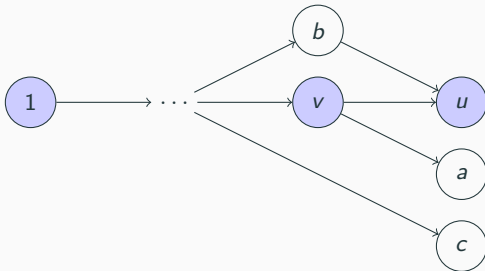
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Construction

Pop vertices with outdegree 0 in this order:



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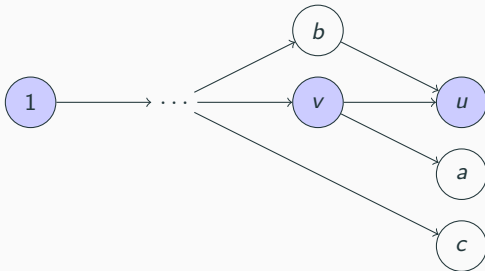
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Pop vertices with outdegree 0 in this order:

- $k - 1$ vertices including everything reachable from v , then u



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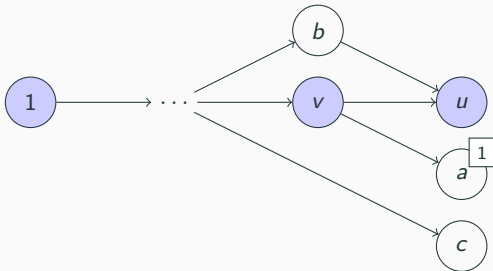
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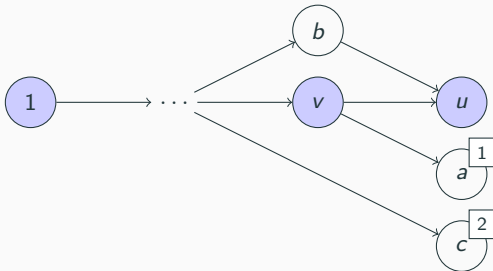
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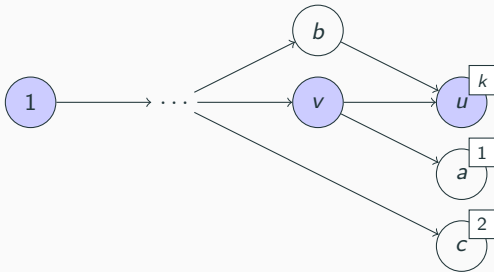
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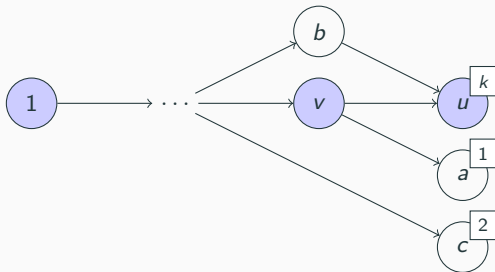
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Pop vertices with outdegree 0 in this order:

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- all remaining vertices, but v is not allowed



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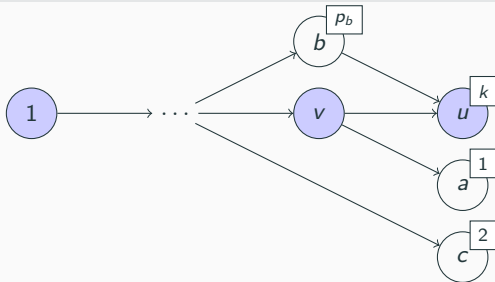
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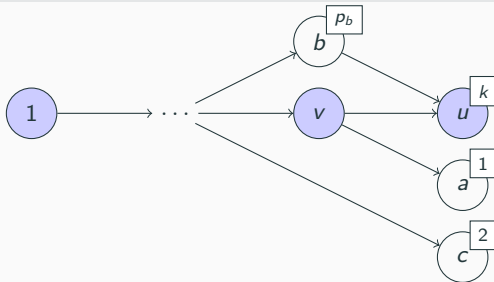
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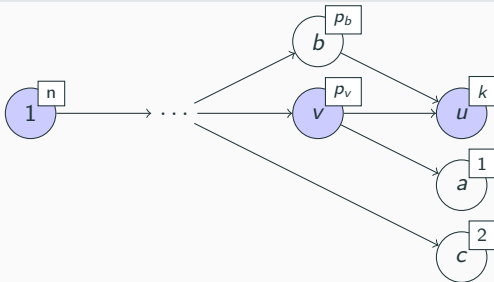
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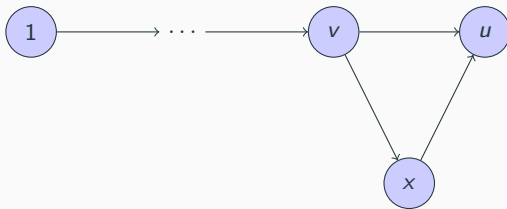


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Note

In the reconstruction, it's possible that v is *not* directly before u .



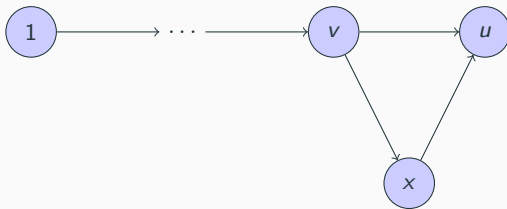
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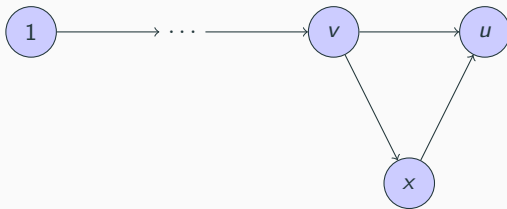
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Note

In the reconstruction, it's possible that v is *not* directly before u .

- Reconstruction is still valid
- Alternatively, among all possible v , pick the one which is “closest” to u .



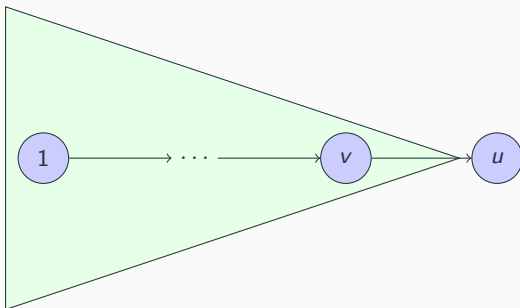
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Proof Idea

- Let S be the set of vertices which can reach v (including v).
- $\min_{a \in S}(p_a) > \max_{b \in V \setminus S}(p_b)$.
- The graph induced by S has only one vertex without outgoing edge: v .

\rightsquigarrow Greedy path contains v .



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Complexity

- For each vertex, count the number of reachable vertices in $O(nm)$
- Check necessary conditions for all edges in $O(m)$
- Reconstruction in $O(m)$

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- Reconstruction in $O(m)$

Possible speedup (not required): bitsets for reachability in $O(\frac{nm}{W})$

Statistics: ... submissions, ... accepted, ... unknown