I: Incremented Itinerary

Problem author: Niklas Mohrin

Problem

Given an undirected graph. Is there a path from vertex 1 to vertex n that is exactly one edge longer than a shortest path?

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Solution 1

- Let $D = \mathrm{dist}(1, n)$.
- **Observation:** Any path from 1 to n of length $D + 1$ must contain an edge $\{u, v\}$ such that the prefix from 1 to u and the suffix from v to n are shortest paths (possibly $u = 1$ or $v = n$).
- Otherwise, the path would have length at least $D + 2$.

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- Otherwise, the path would have length at least $D + 2$.
- Therefore it suffices to check whether there exists an edge $\{u, v\}$ with

$$\mathrm{dist}(1, u) + 1 + \mathrm{dist}(v, n) = D + 1.$$

- To check this efficiently, precompute all distances from 1 and from n using two BFSs.
- Time complexity: $\mathcal{O}(n + m)$.

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Solution 2

- Use two states $(v, 0)$ and $(v, 1)$ per vertex.
- States (u, i) and (v, j) are connected iff there is an edge $\{u, v\}$.

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- States (u, i) and (v, j) are connected iff there is an edge $\{u, v\}$.
- Run a BFS on these states starting from $(1, 0)$, with the following restriction:
  - States $(v, 1)$ can only be visited if reached at distance exactly one longer than the distance to $(v, 0)$.

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- Run a BFS on these states starting from $(1, 0)$, with the following restriction:
  - States $(v, 1)$ can only be visited if reached at distance exactly one longer than the distance to $(v, 0)$.
- Finally, check if $(n, 1)$ is visited.
- Time complexity: $\mathcal{O}(n + m)$.