J: Junior Joining

Problem author: Yidi Zang

Problem

- Given $2n$ recruits with attack a_i, defense d_i, and home city c_i.
- Assign them into n pairs, maximize sum over fighting power.
- Fighting power of a pair (i, j) is $\begin{cases} a_i + d_j, & \text{if } c_i \neq c_j, \\ a_i + d_j + c_i, & \text{if } c_i = c_j \end{cases}$

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Solution

- Instead of pairing, partition recruits into two groups of n, attacking and defending.
- Without city bonus, it is optimal to sort by $a_i - d_i$ and split in half.
- Therefore, for all recruits of one city, sort by $a_i - d_i$.

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- Initially, split each city in half to maximize city bonus.
- If there is an odd number of recruits, ignore the middle one.
- Use middle ones to fill both groups to n.

- For each recruit, calculate attack-defense difference to move to other group.
- Subtract c_i city bonus if they were not middle one.
- As long as it is worth it, swap the best recruit of both groups.
- The resulting partition is optimal, calculate the answer.

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- Whenever we move one (not middle), we lose one city bonus pairing.
- If a recruit from a city c moves from group A to B, no one from c will move from B to A.
- The middle always moves before any other from their city moves.
- Notice that for each city, the recruits are always at an optimal assignment (no swap is better).
- If a better optimal solution were to exist, at least two of different cities are swapped.
- At the end of our process, no two such people can exist.