

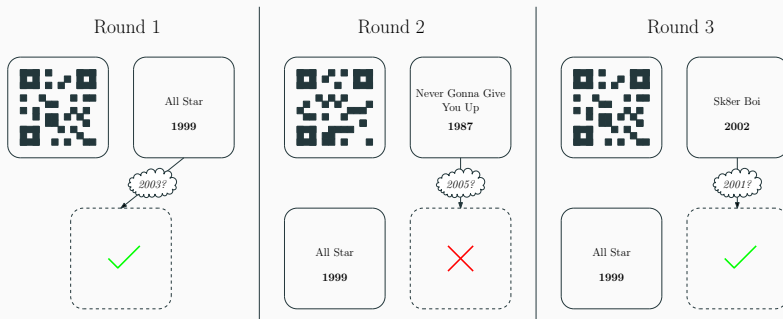
H: Historical Hits

Problem author: Lucas Schwebler

Problem

Find the expected number of correct guesses in a single player game of Hitster (random card order).

- $n \leq 3000$ songs, release year a_i , guess b_i
- a_i are distinct!



Solution

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- Challenge: Solve the problem with duplicate a_i allowed.